Research Statement

My research interests cover a broad area of problems in financial economics with the emphasis on applications of mathematical methods to this area, but recently also encompassing experimental work.

My research can be divided into three areas, which are strongly intertwined:

1. Behavioral finance,
2. Portfolio optimization,
3. Applied mathematics.

1 Behavioral finance

1.1 Prospect Theory and decisions under risk

Financial decisions can only be understood by considering individual behavior and its deviations from rational behavior. Financial markets are based on a multitude of decisions by individuals. Therefore it is only natural that decision theory plays a fundamental role in understanding financial markets. Probably the most important class of financial decisions are decisions under risk. They can be modeled with the celebrated “Cumulative Prospect Theory” by Amos Tversky and Daniel Kahneman (Nobel prize 2002). This theory describes the “subjective utility” of an uncertain event (e.g. an investment) by a functional which (in the most general case) can be written as

\[ CPT(p) := \int_{-\infty}^{+\infty} v(x) \frac{d}{dx} \left( w \left( \int_{-\infty}^{x} dp(x) \right) \right) dx, \]

where \( v: \mathbb{R} \rightarrow \mathbb{R} \) and \( w: [0,1] \rightarrow [0,1] \) are the value and weighting function and \( p \) is the probability measure that reflects the distributions of the possible outcomes.

The study of this model from theoretical, but also from the applied point of view is an interesting field with many open problems.

As a starting point I study in a joined work with Mei Wang (University of Zürich) the classical “St. Petersburg Paradox” by Daniel Bernoulli in the case of Cumulative Prospect Theory [1]. The St. Petersburg Paradox was pivotal in the development of Expected Utility Theory and it turned out that it provides interesting theoretical insights into Cumulative Prospect Theory as well. In particular, we derive sharp conditions on the functions \( v \) and \( w \) under which there exist probability measures with a finite expectation value, but infinite subjective utility.

Although currently Cumulative Prospect Theory has been used in financial economics more frequently than its predecessor Prospect Theory, there is some psychological evidence in favor of the latter theory. The mathematical investigation of Prospect Theory which tries to circumvent some of its theoretical limitations, is therefore another interesting field of research. In particular, since the formulation of this theory in the 1970s, it had been an open problem to extend this theory to general probability measures. In a
joint work with Mei Wang we solve this problem at least for the class of absolutely continuous probability measures by applying convergence methods from functional analysis [2].

Whether one works with Prospect Theory or Cumulative Prospect Theory, in both cases it is possible to quantify the attitudes of individuals towards risk with certain parameters. Since the model can be used to predict, e.g., investment behavior, it is of high interest to compare different groups of individuals with respect to their Prospect Theory parameters. In an ongoing project with Thorsten Hens and Mei Wang [3] we perform this comparison across several countries. Besides this first multi-country comparison of risk attitudes based on Prospect Theory we also compare time-discounting and ambiguity aversion and measure the so-called “cultural dimensions” used by psychologists to capture main cultural features around the world. The analysis of these results will be performed in 2007.

Recently, together with Mei Wang, we also worked on a mathematical analysis of a competing decision model, the so-called “priority heuristic”. We showed why this model seems to fit existing data on decisions under risk surprisingly well, and demonstrated its limitations. Moreover we found a geometrical description of general heuristic models [4].

### 1.2 Connection to game theory

Game Theory and Expected Utility Theory have been presented simultaneously by von Neumann and Morgenstern and are ever since considered to be inseparable. Given, however, the overwhelming evidence on the limitations of Expected Utility Theory as a descriptive model for decisions under risk, it seems quite natural to try to substitute Expected Utility Theory by a behavioral model like Prospect Theory.

In a recent work [5], I demonstrate that this can lead to surprising effects, in particular the potential non-existence of Nash equilibria when considering the “framing effect”. Under certain restrictions on the choice of frame, however, it is possible to prove existence of Nash equilibria. A second effect is the shift of mixed strategy equilibria by probability weighting. As an interesting application I consider “social control games” and prove that a strong probability weighting of individuals leads generally to increasing common wealth when playing such games. Moreover, under weak additionally assumptions, probability weighting turns out to be evolutionary stable. This might provide a potential explanation for the prevalence of probability weighting in individuals.

### 1.3 Behavioral finance aspects of structured products

In an ongoing project with Mei Wang and Patrick Frei, a PhD student at the University of Zürich, we are studying barrier options [6]. Our aim is to prove that these, currently popular, structured products are only interesting for investors with a non-concave utility function or more precisely, with a convex-concave value function as Prospect Theory would predict.

Our numerical simulations support this idea so far very well. We can also show that for reasonable Prospect Theory-parameter different variants of barrier products are preferred. At the moment we want to make some of our results mathematically rigorous by replacing the numerical computations by sharp estimates.
This would be the first result that shows that the success of certain structured products cannot be explained by classical Expected Utility Theory.

2 Portfolio optimization

2.1 Co-monotonicity

What general properties do optimal portfolios have? This seems at first to depend on our underlying decision model, be it rational or behavioral. It turns out, however, that certain properties are independent of the decision model.

Previous work by various authors showed that in very simple cases (e.g. finite state spaces with Expected Utility Theory as decision model) an optimal portfolio has to be co-monotone with the state price density. In the special case of a CAPM market this implies that optimal portfolios “follow the market”, i.e. their performance is the better, the better the market portfolio performs. We are now able to generalize these results into several directions: the result holds for any decision model, for arbitrary state spaces and even when the valuation of the portfolio is measured with respect to a benchmark (which, in the case of CAPM, can be any monotone function of the market return) [7, 8].

The key ideas of the proof rely on results in the context of transport problems as introduced by Kantorovich. Recent results that ensure the existence of monotone transport plans can be applied directly to the conjoint probability distributions between optimal portfolios and the state price density. The underlying mathematical results in [9] had been applied previously to problems totally unrelated to financial economics, namely in fracture mechanics [10].

2.2 Optimal structured products and portfolios

Structured products are in recent years a growing market also for private investors. It is therefore not only of academic interest to ask what products would be optimal, given a certain decision model (e.g. Expected Utility Theory or Prospect Theory).

Using the recent results on co-monotonicity sketched above and no-arbitrage constraints, together with Mei Wang and Thorsten Hens, we develop a method to find such optimal structured products. Our method is not restricted to the combination of certain assets, but instead allows for all outcome distributions with a fixed price, based on the no-arbitrage condition and derives an optimization scheme for optimal financial products [11, 8]. As major mathematical tools we use methods from the calculus of variations, but also transport theory to handle the correlation between market and portfolio. We encounter certain problems when allowing for arbitrary state price densities that give interesting insights into the limitations of the existence of optimal structured products.

Results of our research have been applied in a project with the Zürcher Kantonalbank. Another, more traditional approach, asks for optimal asset allocations among a certain number of assets. Based on their past performance, we want to combine them into a portfolio which optimizes the decision criterion of the investor. Particularly interesting is the case of a behavioral investor who follows Prospect Theory and therefore overweights small probability events. One of the goals is to derive new auxiliary tools for the analysis of assets and the optimization of portfolios, based on a more rigorous
mathematical understanding of the underlying procedure. This is a collaboration with Thorsten Hens and János Mayer (Unviersty of Zürich) [12].

3 Applied Mathematics

3.1 Calculus of Variations and Optimization

In a series of articles with several co-authors I studied variational problems with volume constraints. In such problems we aim to minimize (or maximize) a certain functional among a class of functions such that the volume on which the function admits certain values is prescribed. This class of problems shows a mathematically rich and interesting behavior where conditions for the existence of solutions are often quite delicate, and even in innocent looking cases, non-existence can occur. Besides the work on existence and on methods for the numerical approximation of solutions we also studied the limit where the non-prescribed volume tends to zero [13, 14, 15, 16]. The problem can be generalized by prescribing not only the volume of certain values, but of all values [17]. This specific problem has an interesting connection to portfolio optimization: if we are interested in generating a certain outcome distribution, i.e. a probability measure on the returns, can we define a (reasonably regular) function on a state space, given, e.g., by the returns of several underlyings? It would be interesting to study this connection in more detail in the future.

Another work in the context of Calculus of Variations has already been mentioned, namely the existence of monotone transport plans in the general one-dimensional transport problem [9] with its applications to co-monotonicity of optimal portfolios.

3.2 Multiscale problems

In the past I have also worked extensively on multiscale problems, i.e. problems with scales of very different order of magnitudes. Mathematically, these scales are usually decoupled, in that one studies an appropriate limit where one of them tends to zero. Typical mathematical tools in this area are Young measures and Gamma convergence. Together with several co-authors and students, I applied these concepts to analytical and numerical problems from image recognition (Mumford-Shah functional) [18], micro- and nanostructures of crystals [19, 20, 21, 22, 23] and fracture mechanics [24, 10].

Multiscale problems are, as the list of topics covered in my previous research shows, ubiquitous. I am also aware of applications of Young measures in economics, e.g. in existence results for Nash equilibria. It could be interesting to study the impact of recent results in the field of multiscale analysis on such problems.

Ongoing projects and future plans

One of the projects I am currently involved in is the composing of a text book in financial economics (together with Thorsten Hens) [25]. Our focus is hereby a mathematically solid introduction that is providing the necessary tools to tackle problems of practical
relevance in classical, but also in behavioral finance. The book is expected to appear in 2007.

Another project in economics, albeit not directly connected with finance, that I would like to mention, is an ongoing work with Alexander Wagner (University of Zürich) on optimal inter-temporal budget allocation.

Besides this work and the current projects that I have already discussed in the previous sections, there are several interesting topics I would like to work on in the future:

First, there is the “psychological” or “behavioral” side of the work with Thorsten Hens and János Mayer, mentioned before [12], i.e. the question how individuals consolidate numerous disperse events (or potential outcomes) into a small number of approximate ones, e.g. how individuals transform a set of historic outcomes into a lottery that provides them with a reasonably sized representative sample of potential events. This question is particularly interesting in the context of Prospect Theory, since weighting of probabilities is only defined for lotteries, and it is therefore pivotal to understand the transformation from data to lotteries better.

Related to this question is a more general one: we understand nowadays (thanks to Prospect Theory and related models) the decision process starting from well-defined lotteries reasonably well. But how do people arrive at the lotteries? In a real-life financial decision, people are not provided with simple lotteries, but have to estimate the relevant probabilities by themselves. This effect is very relevant, as I could show in the case of barrier products which are usually sold as knock-out options (i.e. capital protection until the underlying hits a barrier, e.g. at 80% of the original value). Why this? Given the results on co-monotonicity, one can show that this is non-optimal and one can construct a structured product with the same outcome distribution, but a lower price! However, it seems that the knock-out option leads to an underestimation of the (negative) event of hitting the barrier and consequently losing the capital protection, at least this is confirmed by some preliminary experiments with university students.

This example shows: a solid and general theory describing the process of probability estimation in financial decision would be of high relevance and is a source for future research.

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References


